EBS 289 : Psychological Basis of Teaching and Learning Mathematics

Unit 1

Objectives

By the end of the unit, the student will be able to;

- explore and discuss the rationale for teaching and mathematics at the primary and JHS level.
- discuss the various definitions of the psychology of mathematics
- explore the psychological Basis of Teaching and Learning Mathematics.
- discuss the effective ways of teaching and learning mathematics.

As a teacher of mathematics, you do not doubt how multiple students ask what is a very complicated question: "Why do I have to learn mathematics?" At one time or another, perhaps while insistently going through a series of context-free symbolic manipulation exercises, nearly every one of us has struggled to provide our students with an answer that they, and/or we, find satisfactory. But the student "why?" question raises an equally important, but less often-asked question: "Why do we teach math?"

Francis Su, past president of the Mathematical Association of America, is of the view that this is a very important question because how we answer it strongly influences who we think should do mathematics and how we will teach it.

Ernest (2010), emeritus professor of philosophy of mathematics education at the University of Exeter, UK, offered three major reasons (and additional sub-reasons) why we teach mathematics because:

- Mathematics is necessary for life – mathematics for employment and the economy. Ernest included functional numeracy; practical and work-related knowledge; and advanced specialist knowledge under this reason.

- Mathematics has Social and Personal relevance – mathematics for personal and social relevance. Here Ernest included mathematical problem posing and solving; the development of mathematical confidence, including mathematical persistence; and social empowerment through mathematics.

- Mathematics as an Element of Culture -- the importance not only of appreciating mathematics itself, but also its role in history, culture and society in general.

Mathematics education has traditionally emphasized what Ernest labelled "Necessary Mathematics" over other reasons for teaching and learning mathematics. This bias has long historical roots stretching back to the fourteenth century when European mercantile schools first began teaching arithmetic out of an economic need for efficient calculation.

Similarly, arithmetic was added to the curriculum in the American colonies largely in response to the needs of the business. And the current standards-based mathematics education reform effort, which continues to be driven by the discussion of national economic interests and the associated emphasis on college and career readiness, has its roots in national defence and economic concerns stretching back to WWII and the Soviet launching of Sputnik. (For those interested in the history of school mathematics education in the United States, I highly recommend NCTM' s two-volume set, A History of School Mathematics)

As early as An Agenda for Action (1980) NCTM argued that students should learn mathematics for more than economic reasons stating that "all reasonable means should be employed to assure that everyone will have the foundation of mathematical learning essential to fulfilling his or her potential as a productive citizen" (p. 16).

In Principles and Standards for School Mathematics (2000) the Council strongly stated that students need to learn mathematics, and by extension, we teach mathematics, for reasons beyond, but including, "necessary mathematics."

- Mathematics for Life – knowing mathematics can be personally satisfying and empowering.
- Mathematics as Part of Cultural Heritage
- Mathematics for the Workplace
- Mathematics for the Scientific and Technical Community (p. 4)

The recent emphasis on college and career readiness standards has certainly emphasized the latter two reasons over the first two as the primary reasons for teaching and learning mathematics. I admit I have more than once told a student that the reason why they have to learn something is that "they will need it for college" or "the next course." Today I appreciate that this response was lazy on my part, and from my perspective, while critically important, not even the primary reason why we should teach mathematics.

I believe the answer as to why we teach mathematics is in part answered in the NCTM Vision statement. The NCTM vision statement in part states that "We envision a world where everyone is enthused about mathematics, sees the value and beauty of mathematics and is empowered by the opportunities mathematics affords." In this statement, we find an emphasis on the first two reasons for teaching and learning mathematics first offered in Principles and Standards. What does it mean to be empowered by mathematics? In addition to preparing students for careers and post-secondary education, I believe it means we also teach mathematics to equip students for active participation in our democratic society. We accomplish this goal by emphasizing analysis and critical thinking with mathematics so that individuals can identify and interpret claims made by those in power as truthful or false and misleading.

We live in a world where mathematics is increasingly used to characterize societal problems and formulate proposed solutions. Without mathematics literacy, and a strong mathematics identity and sense of agency, members of our society will increasingly find it difficult to comprehend and critique, let alone challenge, many of the decisions and actions of those in power in political, social, scientific, and economic institutions.

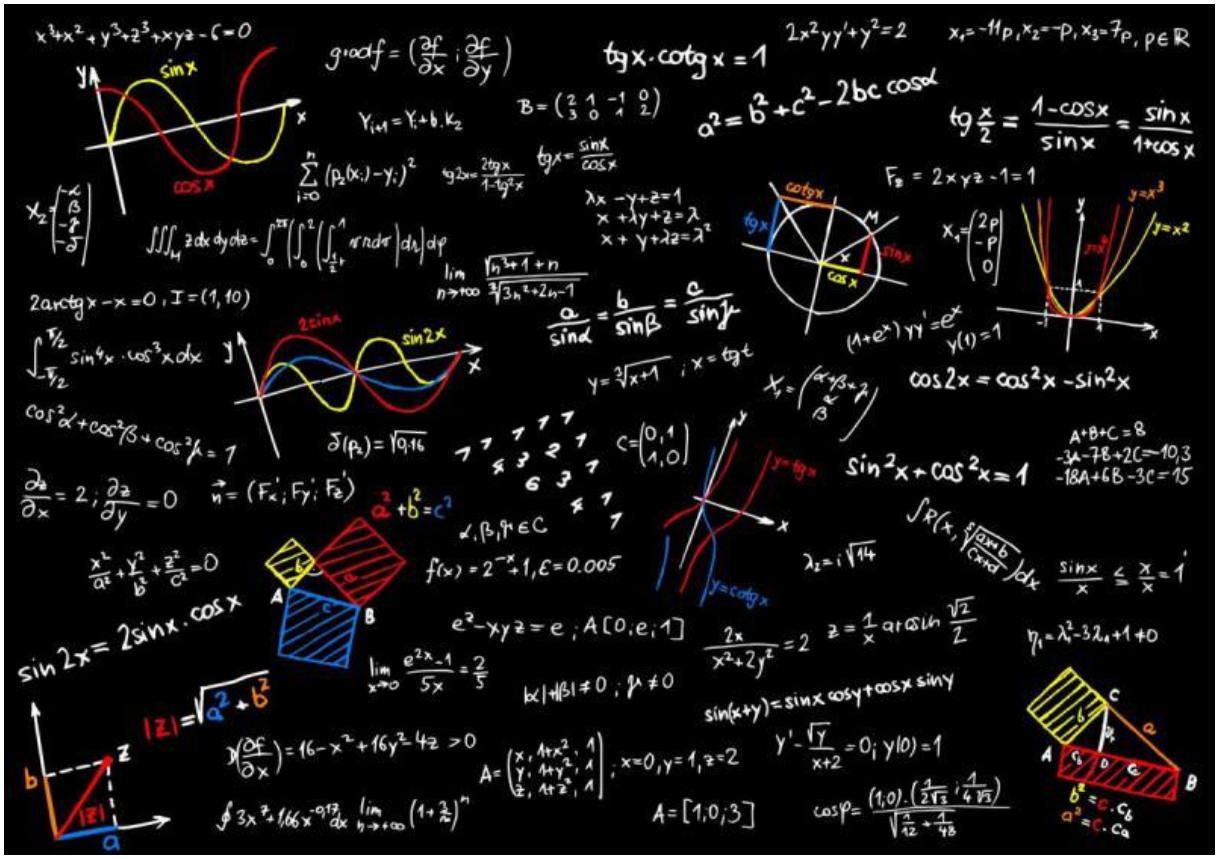
Ultimately, I believe we should teach mathematics (and students should learn mathematics) for multiple reasons – a theme that NCTM will continue to emphasis.

In most cases, this means we need to increase our emphasis on teaching "Mathematics for Life" and "Mathematics as Part of Our Cultural Heritage." Teaching mathematics as part of cultural heritage means that we do **not** just emphasize the dominant culture and heritage. By valuing and developing a better understanding of each other and other cultures, including the multiple contributions various cultures have made to mathematics, we cultivate and nurture student identities. If we teach mathematics so that students are empowered by it, preparation for college and careers will largely take care of itself.

We should never forget, or fail to appreciate, that as teachers of mathematics, each one of us is engaged in something much more important than our daily tasks of instruction, curriculum, and assessment. We are engaged in empowering our students so that they can improve not only their own lives but can also better understand and critique the world around them.

I challenge you and your colleagues to discuss Why do we teach and learn mathematics?

If we start by engaging in critical conversations around these questions, many of the debates in mathematics education and how we resolve them concerning our instructional tasks, our curriculum, and our assessments, will suddenly have a clearer direction.



(Image source)

One could also argue that we have to teach and learn mathematics because the mathematical procedures that are taught in schools will be useful to students later, but I am pretty sure this is false. Almost everyone forgets those procedures as they get older because most people in our society use virtually none of the procedures they learned in school in their day-to-day life. There are engineers, mathematicians, and scientists who use the mathematics they have learned, possibly daily, but I think if you dig deeper into the work they do, many of these people use tools to help to do their work (like Mathematica, for example), look up the finer

details of mathematical procedures that they do not use often, or who use only a very specialized portion of their mathematical knowledge regularly.

It could be that we want to expose students to different ways of thinking about the world. In this case, we would be less concerned with the exact set of mathematical procedures they have learned, and more concerned with learning mathematics as a way of thinking and knowing. I see little evidence that this is an explicit goal of mathematics instruction given that; the students are assessed only on the procedures, teachers are assessed on their students' understandings of those procedures, and that the set of mathematical procedures we want students to know is so prescribed such that it is virtually identical around the world.

It could be that we would like students to learn transferable problem-solving skills. In this case, we want to teach mathematics in such a way as to promote the likelihood that students will be able to transfer what they learn to other areas. The cross-disciplinary study would be the norm, rather than the exception. It turns out that "teaching skills that transfer" is not as simple as one thinks. My understanding is that most of the times when people learn skills in one context, they do not end up transferring those skills to other contexts. Instruction that aims for transferable skills has to provide opportunities for students to make connections between different areas, reflect on what they have learned, and develop metacognitive strategies so that students think about their thinking. What evidence is there that these types of activities are a regular part of math classes?

It could be that we would like students to see the beauty and elegance of mathematics. One way to do this could be through exploring mathematical art. Another might be to look at some famous examples of truly elegant uses of mathematics. We could also ask students to talk about mathematics in the abstract and come to a shared understanding of what elegance and beauty in mathematics mean. As far as I know, none of these activities is a common one in math classes. It is depressing to me that this way of thinking which has so much beauty in it is shared in such a way that almost no one in our society ever gets to experience beautiful mathematics.

If one or more of the reasons I suggested above is something you think is a good reason to teach mathematics, how are you ensuring that you meet this goal with what happens for students in your classroom?

National Council of Teachers of Mathematics (2000) outlined the following Principles and standards for teaching and learning mathematics:

Excellence in mathematics education requires equity – high expectations and strong support for all students.

All students, regardless of their characteristics, backgrounds or physical challenges, can learn mathematics when they have access to high-quality mathematics instruction. Equity does not mean that every student should receive identical instruction. Rather, it demands that reasonable and appropriate accommodations are made and appropriately challenging content be included to promote access and attainment for all students.

A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades.

In a coherent curriculum, mathematical ideas are linked to and build on one another so that students' understanding and knowledge deepen and their ability to apply mathematics expands. An effective mathematics curriculum focuses on important mathematics that will prepare students for continued study and for solving problems in a variety of school, home and work settings. A well-articulated curriculum challenges the students to learn increasingly more sophisticated mathematical ideas as they continue their studies.

Effective mathematics teaching requires an understanding of what students know and need to learn and then challenging and supporting them to learn it well.

Students' understanding of mathematics, their ability to use it to solve problems and their confidence in doing mathematics are all shaped by the teaching they encounter improve in school. To be effective, teachers must understand and be committed to students as learners of mathematics. They must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. Teachers must be supported with ample opportunities and resources to enhance and refresh their knowledge.

Students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge.

Research has solidly established the important role of conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, students can become effective learners. They will be able to recognize the importance of reflecting on their thinking and learning from their mistakes. Students become competent and confident in their ability to tackle difficult problems and willing to persevere when tasks are challenging.

Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

When the assessment is an integral part of mathematics instruction, it contributes significantly to students' mathematics learning. Assessment should inform and guide teachers as they make instructional decisions. The tasks teachers select for assessment convey a message to students about what kinds of mathematical knowledge and performance are valued. Feedback from assessment tasks helps students in setting goals, assuming responsibility for their learning and becoming more independent learners.

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Students can develop a deeper understanding of mathematics with the appropriate use of technology. Technology can help support investigation by students in every area of under mathematics and allow them to focus on decision making, reflection, reasoning, and problem-solving. The existence, versatility, and power of technology make it possible and necessary to re-examine what mathematics students should learn as well as how they can best learn it.

Psychological Foundation of Mathematics

A Pre-service mathematics teacher told his colleague that there is no need for studying the psychological basis of teaching and learning mathematics. What is your take on this?

Preservice teacher's view point is wrong. Studying Psychological Basis of Teaching and learning mathematics has the following merits:

- Mathematics teachers need to study diverse psychologies of learning so that individual learners may be guided to attain maximum learning as optimally as possible. With a thorough knowledge of the psychology of learning, teachers may do a better job of teaching mathematics to pupils of all ability levels. Individual differences among learners will be provided for so that each pupil may learn as much mathematics as possible. A quality mathematics teacher emphasizes objectives, learning opportunities, and appraisal procedures that assist pupils individually to perceive meaning in mathematics learning.
- Knowledge of psychological principles equips the teacher to understand how students learn (that is how the human brain functions). Learning in general deals with the use of intellect and mathematics is an intellectual activity. Teachers need to know-how experience and human thinking interact to bring about mathematical ability.
- The theories of learning explain the individual differences among the learners we teach. We need to rely on sound theories to teach mathematics well to the heterogeneous group of learners we deal within the classroom.
- Theories of psychological principles help the teacher to back the assertion that mathematics should be taught as a process and not as a product so that learners will understand and follow the steps well and enjoy mathematics. The psychological theories give back to the teaching strategies teachers employ in the classroom and this makes the teacher to mathematics.